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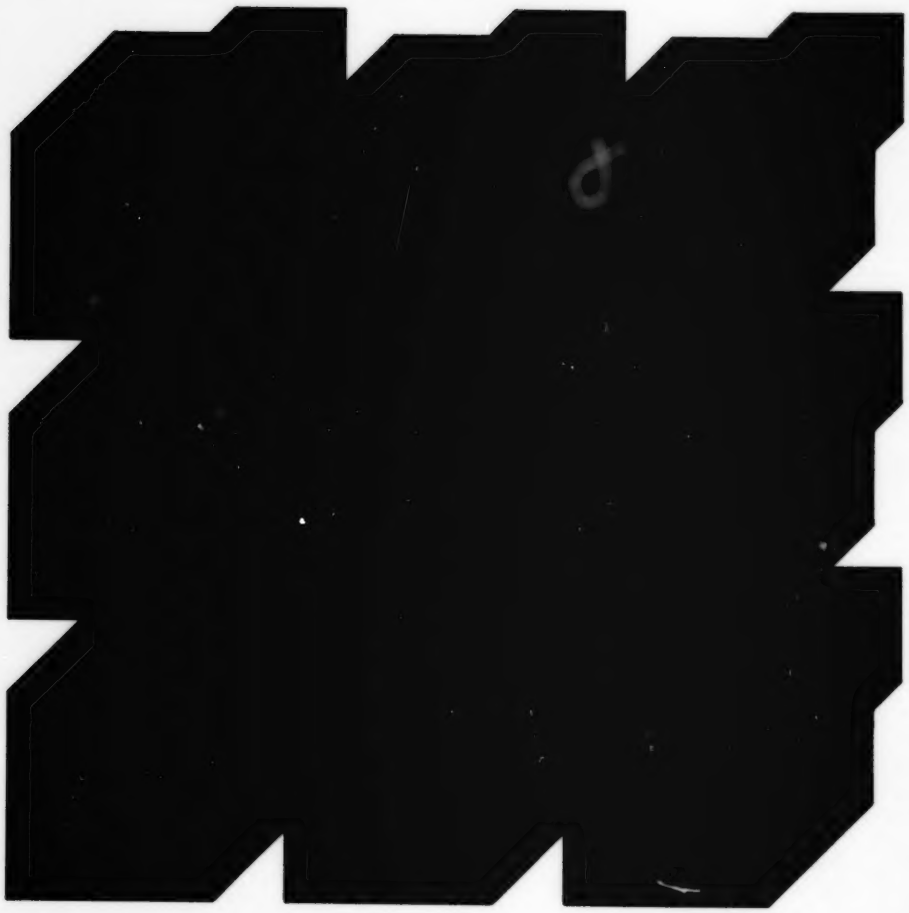

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DISCUSSION PAPER NO. 321

On Calculating Marginal Tax Rates From Stock-Market Prices

by

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and Denis Morissette

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RÉSUMÉ

Les modèles qui ont pour objectif d'évaluer les taux marginaux d'impôt à partir de la variation des cours boursiers lors du détachement d'un dividende ont été relativement populaires aux États-Unis. Nous en examinons deux, proposés par Elton et Gruber (1970) et Auerbach (1981), les appliquons à des données canadiennes et faisons l'analyse des résultats au niveau de l'entreprise. Notre jugement est fondé sur trois critères : l'ordre de grandeur plus ou moins plausible des taux estimés à l'aide des modèles, la stabilité de la clientèle qu'ils prétendent déceler et les différences entre les taux qu'ils attribuent à des groupes différents de contribuables. Les deux modèles échouent aux trois tests. Nous expliquons pourquoi ils ne produiront probablement jamais des estimations fiables lorsqu'on les appliquera à des lois fiscales telles que les canadiennes.

Nos résultats, comme ceux de Miller et Scholes (1982), suggèrent que les estimations "à court terme" que nous avons examinées ne sont pas utiles à ceux qui veulent mesurer l'impact de la fiscalité sur la structure des taux de rendement réalisés par les investisseurs.

SUMMARY

Models designed to infer marginal personal tax rates from price changes on ex-dividend days have been popular in the United States. We examine two of them, one by Elton and Gruber (1970) and the other one by Auerbach (1981), apply them to Canadian data, and examine the disaggregated results they yield. Performance is judged according to three criteria: plausibility of estimated tax rates, stability of tax clientele over time, and differences in rates attributed to stockholders in different tax brackets. Both models fail all three tests. We explain why they will probably never yield robust estimates, especially under tax laws such as the Canadian ones.

Our results support Miller and Scholes' (1982) contention that such short-run measures of the impact of taxes on rates of return are not reliable.

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FOREWORD

This paper is one of the outputs from Council's three year study of the taxation of capital income -- or of the income derived from savings and investment. The study program had important dimensions in both time and space. The effects of capital taxation on both present and future output and standards of living were scrutinized. Taxes levied by all levels of Canadian government were studied as were the international implications of the taxation of capital income. Another important emphasis in the study program was on the interrelationship among specific measures of capital taxation. Here, general equilibrium and other techniques were used to examine the various measures as an interrelated system. Separate studies were also undertaken of specific measures of capital taxation including the personal and corporate income taxes, sales and transaction taxes, property taxes, and resource taxes.

Much of the project research was devoted to finding effective tax rates since these are required to assess the equity and efficiency of the tax system. This paper explores one source of information about effective tax rates on investment income. it attempts to glean from stock market data the rates of tax most probably faced by investors. The authors' method is to infer effective tax rates from the stock market valuation of dividends and capital gains. In this respect, special attention is paid to the 1971 tax reform, which greatly changed the tax treatment of dividends and capital gains.

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INTRODUCTION

Taxes play an important role in financial theory. Maximizing the market value of stocks implies that managers must tailor their policies to the wishes of investors; thus they must consider individual as well as corporate taxes. It follows that financial structure and dividend policies should depend, among other things, on the various provisions of tax laws affecting economic agents. It can also be argued, however, that it is investors who are forced to adapt to corporate decisions. Tax considerations can vary considerably depending on who is investing, and there may in fact be adequate numbers of clients for all possible policy courses. Under these circumstances, the value of a company and its financial decisions may not be tightly linked. Recent research on dividend policy and taxation has been attempting to decide between the merits of these two approaches. The latest work in this area can be divided into two groups.

The first group of studies is concerned with linking dividend policy to the clientele that will find it attractive.[1] In the United States, for example, capital gains are subject to lower tax rates than dividends. Consequently, the total return on a stock, divided into dividends and capital gains, should be directly related to the size of the dividend portion of this return, because of the heavier taxation of dividends. Empirical studies

that have tried to confirm this relationship have run into difficulties of an econometric nature and have failed to produce convincing results.

Others have travelled in the opposite direction.[2] Following Elton and Gruber (1970), some researchers have tried to deduce the marginal tax rate faced by stockholders from the market reaction to a dividend payout. Studies of this type are examined in this paper, which is divided into two parts, corresponding to the two main approaches found in the literature: some have based their work on the Elton and Gruber model (Booth and Johnston (1984), for example), while others, such as Auerbach (1981), have developed alternative techniques based on a capital market equilibrium model.

In each case, the model will first be described and then adapted to the Canadian tax system. Canadian data will then be used to arrive at estimates of tax rates, and their frequency distributions will be examined. Three tests are applied to each model: plausibility of estimated tax rates, stability of the clientele, and differences between rates estimated for different groups of taxpayers. Both models fail all three tests. Finally, we explain why the results obtained are unsatisfactory and conclude that the models do not provide adequate tests of the propositions under examination.

1 THE ELTON AND GRUBER MODEL

1.1 The Model, its Parameters, and Estimation Difficulties

1.1.1 Initial Model

The line of reasoning put forward by Elton and Gruber (1970) is as follows. An investor who holds a stock can choose to sell it cum-dividend at price P_c and pay tax on the resulting capital gain at rate t_g that is the first option. He can also elect to cash in a dividend D, on which tax at rate t_d must be paid, then sell the stock ex-dividend at price P_x that is the second option. In a market in equilibrium, arbitrage profits should not be possible or, equivalently, stock prices should be such that the two options are equivalent. If it is assumed that the price of the stock at the time of purchase was P_o and that the capital gains tax is paid immediately, the two possible transactions will be equivalent when:

$$P_c - t_g (P_c - P_o) = D (1 - t_d) + P_x - t_g (P_x - P_o) \qquad (1)$$

$$\frac{P_c - P_x}{D} = \frac{(1 - t_d)}{(1 - t_g)} \qquad (2)$$

1.1.2 The Canadian Tax System

Equation (2) can be modified to reflect the Canadian Income Tax Act. According to the Act, the amount of the dividend must be grossed up for income tax purposes, following which the taxpayer is granted a tax credit. In addition, only part of the capital gain or loss is taxable,[3] and until 1972 an additional deduction of 10, 15 or 20 per cent was given to investors receiving dividends from mining, oil or gas companies. This deduction is represented here by X, and it has been assigned the maximum value of 20 per cent.

The stockholder receiving a net dividend D gains a net income RN such that:

$$RN = D\,(1 - t_d)$$

$$= D - \gamma\, D\,(1 - X)\,(t_p) + \alpha\, D(1 - X)$$

where t_d = dividend tax rate,

γ = dividend gross-up factor,

α = dividend tax credit,

X = credit for dividends from gas, mining or oil companies,

t_p = marginal tax rate on investor's income.

Thus:
$$t_d = (1 - X)\,(\gamma\, t_p - \alpha) \qquad (3)$$

Let us assume:

L = proportion of taxable capital gain,

t_g = tax rate on capital gain.

Thus:
$$t_g = Lt_p \tag{4}$$

Equation (2) becomes:

$$\frac{P_c - P_x}{D} = \frac{1 - (1 - X)(\gamma t_p - \alpha)}{1 - Lt_p} \tag{5}$$

To take provincial income tax into account, the following figures must be used for the taxation years 1972 to 1982:

$$t_p = t_{pf}(1 + t_{pp}) \tag{6}$$

$$\alpha = \alpha_f (\gamma - 1)(1 + t_{pp}) \tag{7}$$

and

$$t_p = (1 + t_{pp} - \lambda)\, t_{pf} \tag{8}$$

$$\alpha = \alpha_f (1 + t_{pp} - \lambda) \tag{9}$$

for the taxation years 1970 and 1971.

t_{pf} = marginal federal tax rate,

t_{pp} = marginal provincial tax rate, expressed as a percentage of federal tax,

λ = federal abatement for provincial tax,

α_f = federal dividend tax credit.

The values assigned to these parameters for the various years are listed in Table 1-1. Elton and Gruber, as well as Booth and Johnston, decided to use equation (5) to estimate the marginal rate t_p. The ratio forming the left-hand side of the equation can be calculated from stock market prices and the dividend amount, and once the figures for the right-hand side of the equation are computed, it becomes possible to calculate t_p and t_{pf}.

These figures change constantly as new budgets and tax reforms are brought in. It should be noted that the reform made to the Income Tax Act in 1971 changed the numerical values of all the tax parameters mentioned above (see Table 1-1). It is thus to be expected that tax rate estimates will show significant changes at that point in time.

Table 1-1

Taxation Parameters for an Ontario Resident, 1970-82

Year	X	α	γ	L	λ	Maximums		
						t_{pf}	t_{pp}	t_p
1970	0.2	0.200	1.00	0.0	0.28	0.80	0.280	0.800
1971	0.2	0.199	1.00	0.0	0.28	0.80	0.275	0.796
1972	-	0.348	1.33	0.5	-	0.47	0.305	0.613
1973	-	0.348	1.33	0.5	-	0.47	0.305	0.613
1974	-	0.348	1.33	0.5	-	0.47	0.305	0.613
1975	-	0.348	1.33	0.5	-	0.47	0.305	0.613
1976	-	0.348	1.33	0.5	-	0.47	0.305	0.613
1977	-	0.360	1.33	0.5	-	0.43	0.440	0.619
1978	-	0.540	1.50	0.5	-	0.43	0.440	0.619
1979	-	0.540	1.50	0.5	-	0.43	0.440	0.619
1980	-	0.540	1.50	0.5	-	0.43	0.440	0.619
1981	-	0.547	1.50	0.5	-	0.43	0.460	0.628
1982	-	0.503	1.50	0.5	-	0.34	0.480	0.503

Source Revenue Canada, Taxation Statistics, Ottawa, various years.

1.1.3 Estimation Difficulties

Simplicity of the model represented by equation (5) is only apparent. As shown by Booth and Johnston (1984) and explained further below, any or all tax parameters may change depending upon the assumption one wishes to make about the identity and behaviour of the marginal investor, and estimated values of t_p will vary accordingly. Therefore, estimates of marginal tax rates or of the impact of taxes on rates of return obtained through this model may not be reliable. As a matter of fact, this paper shows that they are not.

Ideally, the cum- and ex-dividend prices used in equation (5) should be measured at very short intervals, so that no other information besides the actual dividend payment affects the estimate. General stock market movements and news items relating directly to the company will have a random influence during the period between P_c and P_x. If stock price variations are large in relation to D, it will be difficult to distinguish the effect of D and so to measure the implicit rate t_p. As Miller and Scholes (1982) suggested, observations at monthly intervals are not likely to provide the type of measurements required, and daily observations would be more appropriate to the methodological framework adopted. Paradoxically, it may be that measurement intervals longer than a month (which would give an indication of long-term equilibrium) would be better suited to the problem at hand.

Equation (5), as presented, assumes that capital gains tax is paid immediately; this is not accurate, however. The value of parameter L, which represents the taxable proportion of the capital gain, should be adjusted downward from its maximum of 0.5 according to the delay until the stock is sold and tax is paid. The exact value of L will depend on the discount rate and the number of periods tax payment is delayed. For the years preceding the 1972 tax reform, however, when capital gains were not taxable, our formula is correct as it stands. The value of L for those years is zero.

Equation (5) also assumes that the investor is considered an individual for taxation purposes. This is not necessarily the case. Retirement saving funds, which are not subject to Canadian income tax, are often major stockholders. Also, a company does not pay tax on dividends it receives from another taxable Canadian company, although it must pay tax on capital gains. Finally, the distinction between dividends and capital gains does not apply to economic agents who are stockbrokers by trade. These differences in taxation are such that the estimated rate cannot be predicted accurately: it may be located anywhere between 0 per cent and the maximum marginal rate. For stocks traded on foreign stock exchanges, the relevant marginal investor might even be a foreigner, as suggested by Booth and Johnston (1984).

A more detailed analysis indicates that the relationship between personal tax rates and the financial policies of corporations is not simply that the former influences the latter.[4] The Income Tax Act inextricably ties together debt and dividend policies, on the one hand, and corporate and individual tax rates, on the other. Stock selection will be made on the basis of long-term considerations, and the financial clientele of corporations with identical dividend policies can be very disparate. Thus it may be impossible to obtain reliable estimates of the marginal effect of taxes through short-term measures.

The final reservation about this model concerns the existence of arbitrage profits. Disregarding transaction costs, the left side of equation (5) should equal one. If it is systematically less, as noted by Miller and Scholes and others, certain investors can realize arbitrage profits. Such an obvious profit-making opportunity should not last for very long. However, when the dividend payout effect is lumped together with the impact of news releases made public between two successive transactions, it becomes impossible to predict the value of this ratio.

The range can be determined within which the ratio $R = (P_c - P_x)/D$ must fall, so that it will not be profitable for investors with minimal transaction costs to make deals during the period around the ex-dividend date.[5] For investors who are

tax-exempt or whose capital gains and dividends are subject to the same tax rate, the following two inequalities must be true:

$$-P_c + P_x + D - 2\Omega P' < 0 \qquad (10)$$

$$P_c - P_x - D - 2\Omega P' < 0 \qquad (11)$$

where Ω = percentage transaction cost for purchase or sale of stocks;

$$P' = (P_c + P_x)/2.$$

With some adjustments to (10) and (11), the two can be combined as follows:

$$1 - \frac{2\Omega P'}{D} < \frac{P_c - P_x}{D} < 1 + \frac{2\Omega P'}{D} \qquad (12)$$

If relation (10) is false, an arbitrage profit can be made by buying a stock cum-dividend and selling it ex-dividend. If relation (11) is false, a short-term profit can be realized by simply short-selling a stock cum-dividend and buying it back ex-dividend. In a situation where the constraint given by relation (12) is operative, any estimate of the stockholder's marginal tax rate that uses a model based on stock market reaction to stocks going ex-dividend will be biased. Empirical estimates

offered in the literature have generally ignored transaction costs. If they were included,[6] relation (2) would change as follows:

$$\frac{P_c - P_x}{D} = \frac{(1 - t_d)}{(1 - Q)\ (1 - t_g)} \qquad (13)$$

Relation (13) shows that the effect of transaction costs is to raise the equilibrium value of the ratio $(P_c - P_x)/D$ above the value it would have if transaction costs did not exist.

Those reservations are so important that one may not be convinced that studies of the ex-dividend by behaviour of stock prices have succeeded in segregating dividend, information, and tax effects. Therefore, we propose to re-examine the available evidence, through somewhat different statistical tests.

So far conclusions have been based on aggregate data, i.e., mean rates and changes in mean rates. Rather than simply examining mean rates, we intend to evaluate the output of the model according to three criteria. First, we shall examine the entire frequency distributions of tax rates to find out the proportion of stocks which yield plausible estimates. Second, we shall examine the correlation of successive estimates of tax rates for the same firm: if they are indicators of some kind of tax clientele, they should be positive. Alternatively, a tax "clientele" which is not relatively stable may have little economic significance. Third,

we shall test whether the model assigns different tax rates to different groups of shareholders. Computations are explained in more detail below.

1.2 Empirical Study

1.2.1 Calculation of Personal Rates

Professors Booth and Johnston were kind enough to give us access to a data base[7] that includes information on all dividends over $0.25 paid between 1970 and 1980 by 144 Canadian corporations. The following data are available:

- dividend amount and the ex-dividend date;

- closing rates for the last five days during which the stock was sold cum-dividend. The final price so defined will be designated P_c;

- opening and closing prices on the ex-dividend day. The opening price will be designated P_x;

- value of stock market index at final closing cum-dividend, and at opening and closing ex-dividend.

The marginal tax rate implicit in the price change accompanying those dividend payments was calculated using the following five-step procedure:

1) The variable P_c was adjusted to reflect the value it would have had at the next day's opening on the basis of index movement and the stock's degree of instability. This last statistic, beta, is measured by taking the regression coefficient of the stock's rate of return on that of the market over the 60 months previous to the dividend payout.[8] The adjusted value of P_c will be designated $\bar{P}_c$. The number of observations is thereby reduced to 130, because the betas for 14 of the companies were not available, but, in this way, we control for risk or volatility of the stock.

2) R was calculated according to the formula $R = (\bar{P}_c - P_x)/D$. Because R varies a great deal, we have employed the following filters: the dividend is not included when

$$(P_c - P_x) > \text{maximum variation over last five days; or}$$

$$(P_c - D - P_x) > (0.10)\ (P_c)$$

Admittedly, those filters are arbitrary and bias the computations towards the model by discarding unusually large price changes. Should our results eventually support the Elton and Gruber model, we would have to show this is not due to the

filters. On the other hand, if they don't, our evidence is thereby strengthened.

3) Each of the 130 companies is assigned to one of the four groups below (hereafter called "control groups") on the basis of information provided by Statistics Canada (1980), Financial Post Corporation Service, and Standard and Poor's:

- "public" corporations: companies that do not fit into one of the three following categories;

- subsidiaries: companies controlled by another company, according to Statistics Canada;

- "private" corporations: companies for which a substantial percentage of voting shares is held by one family or group of individuals, according to the records of Financial Post Corporation Service. For the purposes of this study, it was decided that a corporation was "private" when over 10 per cent of voting shares were held by one family;

- interlisted corporations: companies whose stock is also listed on an American exchange, according to Standard and Poor's Compustat data base.

4) For each dividend payout, we have calculated t_{pf} using the

variable R (calculated above) and parameters on the right side of equation (5) (figures given in Table 1-1). Since the number of rates calculated equals the number of dividends, the annual average value of t_{pf}, designated $\bar{t}_{pf}$, is computed for each year and each company. (In the case of subsidiaries, it is assumed that the dividend is non-taxable and that capital gains are taxable.)

5) Finally, the frequency distribution of $\bar{t}_{pf}$ was computed for each corporation category and for each year. The results appear in tabular form in Appendix A and are summarized in Table 1-2.

1.2.2 Analysis of Results

First test: plausibility of estimated tax rates

The empirical results presented in Table 1-2 provide convincing evidence of the model's inadequacy, at least in its present form. For every year, 50 per cent or more of estimated rates fall into the category of "unacceptable," i.e., tax rates that are negative or above 60 per cent, the maximum marginal rate. For five of the eleven years under study, such unacceptable estimates actually exceed 70 per cent. Remember that these figures were derived using the filters mentioned in step 2. The frequency

Table 1-2

Summary of Frequency Distributions of Calculated Rates for $\bar{t}_{pf}$, 1970-80

Year	Non-calculated rates	$\bar{t}_{pf} < 0$		$0 < \bar{t}_{pf} \leq 0.59$		$0.59 < \bar{t}_{pf}$	
		Number	Per cent*	Number	Per cent	Number	Per cent
1970	49	14	0.17	18	0.22	49	0.60
1971	50	12	0.15	16	0.20	52	0.65
1972	46	13	0.15	24	0.29	47	0.56
1973	38	13	0.14	28	0.31	51	0.55
1974	30	20	0.20	29	0.29	51	0.51
1975	29	13	0.13	26	0.26	62	0.62
1976	24	14	0.13	38	0.36	54	0.51
1977	19	13	0.12	54	0.47	44	0.40
1978	25	12	0.11	43	0.41	50	0.48
1979	13	16	0.14	49	0.42	52	0.44
1980	17	14	0.12	41	0.36	58	0.51

* Percentage showing number of corporations in category in relation to number of rates estimated.

Source Appendix A.

distributions shown are bimodal. The two modes fall into the category of unacceptable rates, which is concealed by average rates and the average of rates. Thus mean rates obtained in this way and published as output of the Elton and Gruber model can hardly be seen as "typical" of figures which are themselves implausible estimates of tax rates.

These results can be attributed to "noise" in the statistical series, as discussed earlier. The figure representing the dividend is generally low in relation to the average daily variation in the price of the corresponding stock. Consequently, tax rate estimates are very sensitive to price fluctuations and so are often amplified to the point where the series comprising four annual rates is thrown out of balance. We explain in Part 2 why this problem is likely always to be present.

Second test: correlation of estimated rates

The objective of the Elton and Gruber model is to estimate the marginal tax rate of the marginal investor. However, it is often suggested that different securities may attract different tax clienteles. (That is why our first test took into account disaggregated data.) One would expect such a clientele to be relatively stable. If it changes entirely from one dividend payment to the next, the concept has no economic content. Therefore, we computed the rank correlation coefficient of

successive tax rates attributed to each stock. For the 10 pairs of years from 1970 to 1980, none of the correlation coefficients even came close to being statistically significant, and four were negative. Thus, the model also fails this second test.

Third test: tax rates by control group

If figures presented in the literature as estimates of marginal tax rates are actually so, they might be related to the distribution of shares among shareholders-investors. For instance, it is not unreasonable to assume that the marginal investor of a private corporation has a higher marginal tax rate than the marginal investor in a public corporation. The marginal tax rates should also differ for subsidiaries and interlisted corporations. We tested, through a chi-square test, the hypothesis of a statistical relationship between "type of corporation" and the frequency distribution of estimated tax rates. Our results are reported in Tables A-1 through A-11. Except for the year 1970, rows and columns of those tables appear to be statistically independent, which is inconsistent with the hypothesis that type of corporation and tax rate are related.

This third test may not be very strong. It assumes that the control group is a proxy for the marginal shareholder. This may not necessarily be so. However, this result does add to the

cumulative evidence that the "tax rates" provided by the model do not seem to be related to any tax-related characteristic.

The Tax Reform of 1972

Although the means of the estimated tax rates, not to mention the rest of the distributions, do not seem to have economic significance, Lakonishok and Vermaelen (1983) have argued that changes in that mean convey information about the impact of income taxes. To test this hypothesis, we compared entire frequency distributions -- not just the means -- of tax rates before and after the tax reform.

The table in Appendix B shows the distributions of tax rates for 56 companies for which the data allow rates to be calculated for each year. The distributions were compared in pairs using chi-square tests, and the results are given in Table C-1. It can be noted that the 1970 and 1971 distributions do differ from those subsequent to 1974. Although far removed from the reform period, however, there are many significant differences. These results suggest the two following hypotheses. First, it is possible that the differences in the frequency distributions can be explained by a change in the response of investors to dividend payments, as a result of changes in tax rules. But, second, it is also conceivable that they are simply due to the changes in the numerical values of tax parameters reported in Table 1-1 and used

in step 4 of the computations described above. An examination of the frequency distributions of ratio R indicates that the second hypothesis is more likely. As shown in Table C-2, there are no significant differences in distributions of R before and after the tax reform. Thus differences that appear in the series of rates are spurious -- they can only be attributed to changes in the tax parameters involved in computing rates, and do not reflect changes in stock prices behaviour in response to dividend payments.

These results are inconsistent with those reported by Booth and Johnston (1984) and Lakonishok and Vermaelen (1983). The effects of the tax reform are not evident when personal tax rates are inferred from short-run changes in stock pieces.

1.3 Conclusion of Part One

There are three conclusions that can be drawn from this first section. First, the Elton and Gruber model cannot, at least for Canadian capital markets, provide acceptable estimates of the marginal tax rates affecting the stockholders of a company. Second, it does not provide evidence that the 1972 tax reform did noticeably affect the response of capital markets to dividend payments. Third, empirical studies such as Harris et al. (1983), which claim to confirm the theories advanced by M. Miller (1977) on the relationship between debt-asset ratios and individual tax rates, cannot be generalized. Even if their results are valid for the United States, which is doubtful, they are not so for Canada.

2 THE AUERBACH MODEL

2.1 The Model and its Parameters

Auerbach's (1981) is a version of the traditional Capital Asset Pricing Model incorporating personal and corporate taxes. It assumes a risk-free asset, homogeneous expectations and prohibition of short sales. The analysis is summarized by the following one-equation model:

$$g_{it} = b_{0i} + b_{1i}d_{it} + b_{2i}r_{mt} + b_{3i}r_{ft} + \varepsilon_{it} \qquad (14)$$

where g_{it} = capital gain per dollar of stock i on the ex-dividend day t = $(P_{x,i}/P_{c,i}) - 1$;

d_{it} = dividend per dollar to which the owner of stock i is no longer entitled as of the ex-dividend day t = $(D_i/P_{c,i})$;

r_{mt} = market rate of return on the ex-dividend day t;

r_{ft} = rate of return on the risk-free asset on the ex-dividend day;

b_{0i}, b_{1i}, b_{2i}, b_{3i} = parameters estimated using a linear multiple regression for each firm i;

ε_{it} = random residual term where $E(\varepsilon_{it}) = 0$ and

$$Cov(\varepsilon_{it}, \varepsilon_{it-1}) = 0, \forall_i, t.$$

It is expected that the relationship of g_i to the various independent variables will be as follows:

$$\delta g_i/\delta d_i < 0, \quad \delta g_i/\delta r_m > 0, \quad \delta g_i/\delta r_f > 0$$

Besides the dividend itself, Auerbach's approach relies on the market rate of return, the risk-free rate of return and the beta coefficient in order to explain the return on stock i on ex-dividend days.

Estimated values of b_0 should not deviate significantly from zero. The b_2 values should theoretically correspond to the beta values of the firms.

Of the various parameters that appear in equation (14), b_1 is particularly interesting, because, according to this model, it is the estimated value of b_{1i} that is used to infer the marginal tax rate of firm i stockholders. Which relation is used to deduce the particular tax rate from $b_1$depends on the assumption made about the effective tax rate on capital gains. If it is assumed that $t_g > 0$, we obtain:

$$-b_1 = \frac{(1 - t_d)}{(1 - t_g)} \tag{15}$$

In the Canadian context, this equation becomes:

$$-b_1 = \frac{1-\gamma t_{pf}\ (1 + t_{pp}) + \alpha}{1 - [t_{pf}(1 + t_{pp})/2]} \tag{16}$$

If it is assumed that $t_g = 0$, we get:

$$-b_1 = (1 - t_d) \tag{17}$$

For Canada, relation (17) becomes as follows:

$$-b_1 = 1 - \gamma\ t_{pf}(1 + t_{pp}) + \alpha \tag{18}$$

2.2 Empirical Study

Equilibrium values of t_{pf} for the years from 1972 to 1980 are obtained by substituting for the relevant parameters in equations (17) and (18).

2.2.1 Data and Methodology

As we did for the Elton-Gruber model, we have used Booth and Johnston's (1984) data bank. Our methodology involves the following steps:

(1) Tax rates were estimated for the 1972-76 and 1978-80 periods. The choice of these particular subperiods is motivated by the relative stability of the tax parameters (see Table 1). For purposes of regression analysis, all firms for which less than nine observations were available during either period were eliminated. There were 58 firms in the 1972-76 period that met this condition, and 52 in the 1978-80 period. The maximum number of yields available for companies paying quarterly dividends was 20 for the first period and 12 for the second. In order to minimize the impact of factors other than stocks going ex-dividend, opening prices on the first ex-dividend day and closing prices on the last cum-dividend day were used to estimate g_{it}.

(2) For all firms meeting the stipulations described below, ordinary least square, multiple regressions as described by equation (14) were performed in order to arrive at an estimate of b_{1i} for any given period.

(3) For each firm and each period, two values of t_{pf} were estimated: first, t_{pf} was inferred from the assumption that $t_g > 0$ (see equation (16)). Secondly, the computation was redone, supposing $t_g = 0$ (see equation (18)).

(4) Again, filters were used: a dividend is disregarded if

$$| -g_{it} - d_{it} + r_{mt} | > 0.02 \qquad (19)$$

The number of companies for regression purposes is thereby reduced to 55 for the 1972-76 period and to 36 for the 1978-80 period. We shall present results obtained both with and without filters.

Our methodology differs in some respects from that used by Auerbach in the United States. First, we used the opening price on the ex-dividend day whereas he used the closing price. As noted by Booth and Johnston (1984), there is no justification for using the closing ex-dividend day price in the Canadian context; this only increases the probability of irrelevant information to become available and increase noise. A second difference is that, in contrast to Auerbach, we used no observations other than those taken on ex-dividend days because daily observations were not available. There is a precedent for this approach. Green (1980), for example, tested a model similar to Auerbach's and only used data on relative returns on ex-dividend days. We now proceed with our three tests. They differ from those of Part 1: the Elton and Gruber model provided one tax rate for each security and each dividend payment; the Auerbach model provides one tax rate for each security and each subperiod.

2.2.2 Empirical Results

First test: plausibility of tax rates

As the Auerbach model provides us with estimates of b_1, from which marginal tax rates can be inferred, we start with summary measures of the former, where b_1 is the mean over all firms for each subperiod. Assuming that Cov $(b_{1i}, b_{1j}) = 0$, $\forall_{i \neq j}$, we also compute:

$$\sigma(\bar{b}_1) = \frac{1}{N} \left[\sum_{i=1}^{N} \text{Var}(b_{1i}) \right]^{\frac{1}{2}} \qquad (20)$$

Our first results are shown in column 1 of Table 2-1. We first note that, when filters are not used, $\bar{b}_1$ has the wrong sign: we have $\bar{b}_1 > 0$ while the model suggests $\bar{b}_1 < 0$. Figures of column 1 should be compared with the ranges of b_1 numerical values necessary for the estimate of t_{pf} to fall between 0 and 50 per cent or, at least, between 0 and 100 per cent. Such figures are provided in Table 2-2. Unfortunately, virtually all the estimates of Table 2-1 (column 1) fall outside the required range.

On the other hand, when filters are used, t_{pf} based on $\bar{b}_1$ become more plausible. However, we have to note that they are quite sensitive to the assumption made about the effective rate of the capital gains tax. Above all, we have to emphasize that even plausible means hide the fact that the majority of their component

Table 2-1

Primary Characteristics of b_1 Values and Estimated Tax Rates

Period	Mean values σ (b_1) 1	Assumption: $t_g = 0$			Assumption: $t_g > 0$			Number of observations 8
		Implied $\bar{t}_{pf}$ 2	Per cent of rates		Implied $\bar{t}_{pf}$ 5	Per cent of rates		
			Impossible[1] 3	Improbable[2] 4		Impossible[1] 6	Improbable[2] 7	
1972-76 (no filter)	0.769* (0.483)	1.217	67.2	87.9	0.944	51.7	93.1	58
1978-80 (no filter)	0.092 (0.562)	0.756	63.4	80.8	0.733	53.8	86.5	52
1972-76 (with filter)	-0.664 (0.340)	0.393	61.8	80.0	0.523	50.9	85.4	55
1978-80 (with filter)	-0.312 (0.446)	0.569	58.3	69.4	0.635	41.7	75.0	36

* Indicates that, whatever assumption is made about the effective tax rate on capital gains, the $\bar{b}_1$ value differs significantly at the 1 per cent level from $\bar{b}_1$ values that generate plausible tax rates. The number below each $\bar{b}_1$ represents its standard deviation.

1 i.e., values of t_{pf} that are negative or greater than one.

2 i.e., values of t_{pf} that are negative or greater than 0.50.

Table 2-2

b_1 Values with Implicit Tax Rates Between 0 and 0.50 or between 0 and 1

Period	Assumption: $t_g = 0$		Assumption: $t_g > 0$	
	b_1 values implying $0 \leq t_{pf} \leq 0.50$	b_1 values implying $0 \leq t_{pf} \leq 1$	b_1 values implying $0 \leq t_{pf} \leq 0.50$	b_1 values implying $0 \leq t_{pf} \leq 1$
1972-76	$-1.348 \leq b_1 \leq -0.478$	$-1.348 \leq b_1 \leq 0.392$	$-1.348 \leq b_1 \leq -0.709$	$-1.348 \leq b_1 \leq 1.128$
1978-80	$-1.540 \leq b_1 \leq -0.460$	$-1.540 \leq b_1 \leq 0.620$	$-1.540 \leq b_1 \leq -0.719$	$-1.540 \leq b_1 \leq 2.214$

observations are not acceptable. Columns 3, 4, 5, and 6 of Table 2-1 show that the proportion of impossible or improbable rates goes from 41.7 to 85.4 per cent, even when filters are used. The frequency distributions of t_{pf} are presented in Table 6. Such disaggregated data are more convincing, because they allow each firm to have its own marginal investor and provide information which is lost when the mean is computed. Examination of entire frequency distributions shows that only a minority of so-called tax rates are actually located in the class intervals where they have to be. In general, observations do not cluster around the mean, but in the tails of the distributions.

To summarize our discussion so far, we note that most estimates provided by the Auerbach model are not plausible, they are dependent upon the assumption about the capital gains tax and the use of arbitrary filters which bias the computations in favour of the model.

Second test: correlation of tax rates

As in Part 1, we tested the hypothesis that the tax clientele of a given firm should be relatively stable. If this is true, the classification of firms according to t_{pf}, for the two subperiods, should remain stable.

Table 2-3

Distribution of Federal Tax Rates Calculated According to Auerbach Model

Period	Intervals of t_{pf}						
	< 0	0-25	25-50	50-75	75-100	> 100	Total
1972-76 (no filter, $t_g = 0$)	15	3	4	5	7	24	58
1972-76 (no filter, $t_g > 0$)	9	1	3	8	16	21	58
1978-80 (no filter, $t_g = 0$)	14	5	5	7	2	19	52
1978-80 (no filter, $t_g > 0$)	7	1	6	12	5	21	52
1972-76 (with filter, $t_g = 0$)	21	6	5	5	5	13	55
1972-76 (with filter, $t_g > 0$)	16	4	4	8	11	12	55
1978-80 (with filter, $t_g = 0$)	9	8	3	3	1	12	36
1978-80 (with filter, $t_g > 0$)	5	4	5	5	7	10	36

Since converting regression coefficients to rates through tax parameters can distort the figures, however, the hypothesis was tested using the b_1 values. It was expected that the rank correlation coefficient between those figures for the two subperiods would be positive.

As revealed in Table 2-4, the Spearman rank correlation coefficient r_s, without filter, diverges significantly from 0 at the 5 per cent level. However, it is negative, instead of having the expected positive sign. If the filter is used, the value of r_s is positive, but not significant. Thus the hypothesis theory that the b_1 coefficients of one corporation as estimated by the Auerbach model remain relatively stable from one period to the next must be rejected.

Third test: tax rate by control group

We repeated the third experiment previously conducted with the Elton and Gruber model. As the results are quite similar, we shall not report them in detail. The frequency distributions of tax rates attributed by the Auerbach model to public, private, interlisted, and subsidiary corporations are not significantly different. On the other hand, Booth and Johnston (1984) have argued that the means are different, at least for interlisted stocks. This suggests two hypotheses, which are not mutually exclusive: either means are not good proxies for the frequency distributions involved, or the model does not provide reliable estimates of marginal tax rates. We already know the first proposition is true and examine the second one in Part 3.

Table 2-4

Spearman's Rank Correlation Coefficients between Estimates of Coefficient b_1 for Different Subperiods

Period	Value of r_s	Number of observations
1972-76 and 1978-80 (no filter)	-0.269* (-1.975)	52
1972-76 and 1978-80 (with filter)	0.141 (0.818)	35

* In the case of a unilateral test, indicates a coefficient significant at the 5 per cent level. The number below r_s is the value of Student's t-statistic.

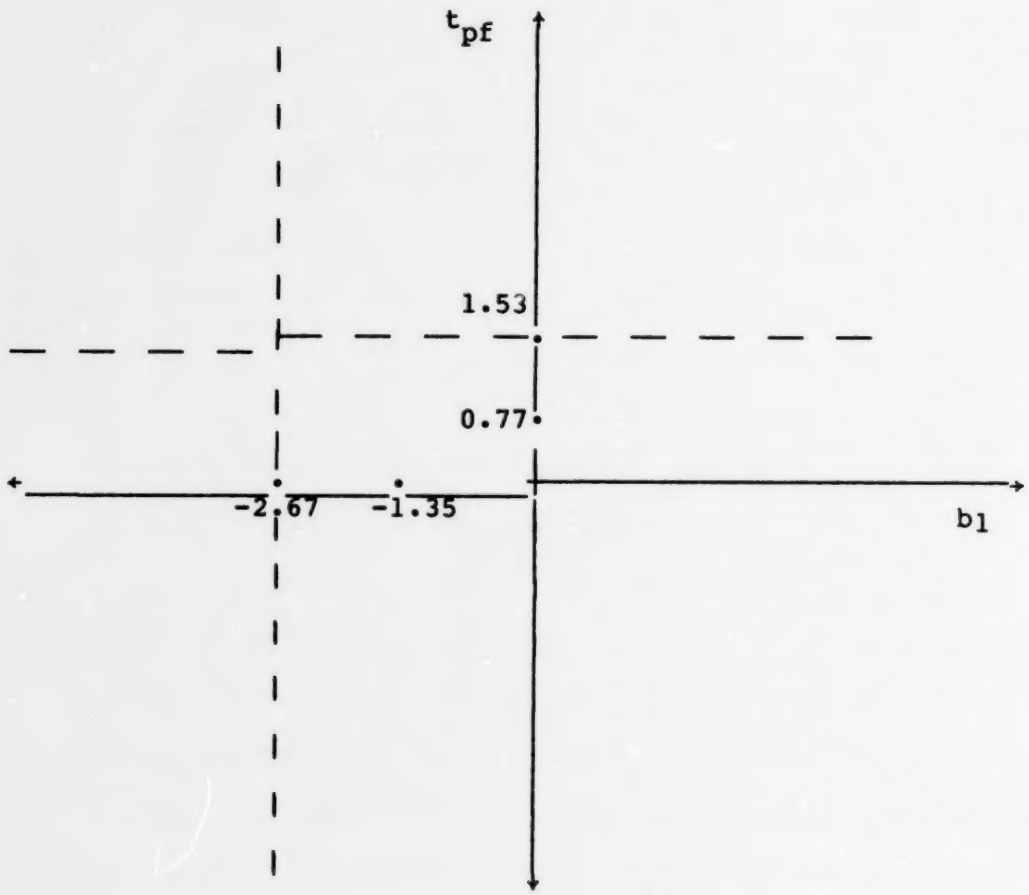
3 TENTATIVE EXPLANATION

The disappointing results of the previous sections are due to the joint occurrence of two elements, such that estimates of tax rates obtained through the Elton and Gruber and Auerbach models will never be robust: the nature of the function relating R to t_{pf} and the abnormal volume of transactions around ex-dividend days. We examine them in turn.

Equations (5) and (16), which summarize the models under examination under Canadian laws, are rational equations. For instance, Figure 3-1 shows the inferred tax rate t_{pf} as a function of b_1 for the 1972-76 period when it is assumed that the effective capital gains tax is greater than zero.[9] In this case, estimates of t_{pf} are realistic only when b_1 falls between -1.35 and -0.71 (see Table 2-2 for other periods). Outside this range the elasticity of t_{pf} with respect to b_1 becomes very large: infinity is the limit of this function.[10] This explains why our filters had such a beneficial effect on estimates: they eliminated large variations in b_1. But it also implies that estimates of b_1 have to be accurate and stable. Otherwise, implicit tax rates will be outside the range of plausible values. (This problem did not arise before the tax reform because there was no capital gains tax.)[11]

Figure 3-1

t_{pf} as a Function of Coefficient b_1 ($t_g > 0$, 1972-76)



Unfortunately, we cannot expect conditions necessary for an accurate estimate of b_1 to occur. As Booth and Johnston (1984) have observed (see also Eades et al., 1984), an abnormally high volume of transactions takes place on ex-dividend days. Large volumes are generally accompanied by large price changes. Thus, the ratio R will have relatively large values on ex-dividend days. This will cause estimates of marginal tax rates to take extreme and unreasonable values. This applies to all tax laws which have to be summarized by rational equations of the type we have used here.

4 OVERALL CONCLUSION

The purpose of this study was to evaluate the performance of certain models designed to infer marginal personal tax rates from ex-dividend day price changes. This evaluation was made using data from the Toronto Stock Exchange; and "filters" were used in order to bias the computations in favour of the models.

Despite these measures, both models fail all three tests: the figures obtained appear to be unusable. Over half of the rates so calculated are improbable, and they do not reflect the differences in fiscal status between public, private, and subsidiary corporations. Moreover, they vary a great deal from one period to another, even for one particular company. In certain cases, however, the means of the distributions do happen to fall within the range of plausible rates. Such mean values should only be used with extreme caution, because the underlying distributions vary widely, are generally asymmetric and bimodal, and are in all cases abnormal. The majority of component observations cannot be interpreted as tax rates.

Thus we feel that the usefulness of the models tested is very limited, at least as far as Canada is concerned. Specifically, this approach leaves unanswered the question of the impact of the Tax Reform of 1972 on the structure of rates of return. Any changes observed between inferred rates before and after this

event can only be attributed to the calculation formulas, and do not reflect changes in the response of stock prices to dividend payments. Our general conclusion is that our knowledge of the impact of tax laws on capital markets based on short-term models rests on shaky ground. Ours supports Miller and Scholes' (1982) conclusion.

APPENDIX A

Table A-1

Distribution of Individual Tax Rates by Type of Corporation, 1970

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public							
freq.	11	4	0	0	1	22	27
Col. %		28.57	0.00	0.00	20.00	44.90	
Row %		14.81	0.00	0.00	3.70	81.48	
Subsidiaries							
freq.	15	4	1	5	1	12	23
Col. %		28.57	16.67	71.43	20.00	24.49	
Row %		17.39	4.35	21.74	4.35	52.17	
Private							
freq.	15	3	2	0	1	1	6
Col. %		21.43	33.33	0.00	0.00	2.04	
Row %		50.00	33.33	0.00	0.00	16.67	
Interlisted							
freq.	8	3	3	2	3	14	25
Col. %		21.43	50.00	28.57	60.00	28.57	
Row %		12.00	12.00	8.00	12.00	56.00	
Total	49	14	6	7	5	49	81
Proportion		17.28	7.41	8.64	6.17	60.49	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	19.292	with 6 D.O.F.	PROB = 0.0037
Cramer's V	0.345		
Chi-square likelihood ratio	18.295	with 6 D.O.F.	PROB = 0.0055

Table A-2

Distribution of Individual Tax Rates by Type of Corporation, 1971

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public freq.	11	2	0	1	5	19	27
Col. %		16.67	0.00	25.00	45.45	36.54	
Row %		7.41	0.00	3.70	18.52	70.37	
Subsidiaries freq.	14	4	0	2	4	14	24
Col. %		33.33	0.00	50.00	36.36	26.92	
Row %		16.67	0.00	8.33	16.67	58.33	
Private freq.	16	1	1	0	1	2	5
Col. %		8.33	100.00	0.00	9.09	3.85	
Row %		20.00	20.00	0.00	20.00	40.00	
Interlisted freq.	9	5	0	1	1	17	24
Col. %		41.67	0.00	25.00	9.09	32.69	
Row %		20.83	0.00	4.17	4.17	70.83	
Total	50	12	1	4	11	52	80
Proportion		15.00	1.25	5.00	13.75	65.00	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	6.305	with 6 D.O.F.	PROB = 0.39
Cramer's V	0.199		
Chi-square likelihood ratio	6.491	with 6 D.O.F.	PROB = 0.3705

Table A-3

Distribution of Individual Tax Rates by Type of Corporation, 1972

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public							
freq.	11	5	3	5	0	14	27
Col. %		38.46	50.00	50.00	0.00	29.79	
Row %		18.52	11.11	18.52	0.00	51.85	
Subsidiaries							
freq.	14	3	2	3	2	14	24
Col. %		23.08	33.33	30.00	25.00	29.79	
Row %		12.50	8.33	12.50	8.33	58.33	
Private							
freq.	14	1	0	0	2	4	7
Col. %		7.69	0.00	0.00	25.00	8.51	
Row %		14.29	0.00	0.00	28.57	57.14	
Interlisted							
freq.	7	4	1	2	4	15	26
Col. %		30.77	16.67	20.00	50.00	31.91	
Row %		15.38	3.85	7.69	15.38	57.69	
Total	46	13	6	10	8	47	84
Proportion		15.48	7.14	11.90	9.52	55.95	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	1.399	with 6 D.O.F.	PROB = 0.9659
Cramer's V	0.091		
Chi-square likelihood ratio	1.377	with 6 D.O.F.	PROB = 0.9672

Table A-4

Distribution of Individual Tax Rates by Type of Corporation, 1973

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public							
freq.	10	4	0	3	3	18	28
Col. %		30.77	0.00	23.08	23.08	35.29	
Row %		14.29	0.00	10.71	10.71	64.29	
Subsidiaries							
freq.	9	3	1	5	3	17	29
Col. %		23.08	50.00	38.46	23.08	33.33	
Row %		10.34	3.45	17.24	10.34	58.62	
Private							
freq.	11	2	0	0	1	7	10
Col. %		15.38	0.00	0.00	7.69	13.73	
Row %		20.00	0.00	0.00	10.00	70.00	
Interlisted							
freq.	8	4	1	5	6	9	25
Col. %		30.77	50.00	38.46	46.15	17.65	
Row %		16.00	4.00	20.00	24.00	36.00	
Total	38	13	2	13	13	51	92
Proportion		14.13	2.17	14.13	14.13	55.43	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	6.849	with 6 D.O.F.	PROB = 0.3351
Cramer's V	0.193		
Chi-square likelihood ratio	7.147	with 6 D.O.F.	PROB = 0.374

Table A-5

Distribution of Individual Tax Rates by Type of Corporation, 1974

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public freq.	8	6	3	1	4	16	30
Col. %		30.00	37.50	16.67	26.67	31.37	
Row %		20.00	20.00	3.33	13.33	53.33	
Subsidiaries freq.	7	7	1	2	4	17	31
Col. %		35.00	12.50	33.33	26.67	33.33	
Row %		22.58	3.23	6.45	12.90	54.84	
Private freq.	10	3	1	1	1	5	11
Col. %		15.00	12.50	16.67	6.67	9.80	
Row %		27.27	9.09	9.09	9.09	45.45	
Interlisted freq.	5	4	3	2	6	13	28
Col. %		20.00	37.50	33.33	40.00	25.49	
Row %		14.29	10.71	7.14	21.43	46.43	
Total	30	20	8	6	15	51	100
Proportion		20.00	8.00	6.00	15.00	51.00	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	1.878	with 6 D.O.F.	PROB = 0.9306
Cramer's V	0.097		
Chi-square likelihood ratio	1.807	with 6 D.O.F.	PROB = 0.9366

Table A-6

Distribution of Individual Tax Rates by Type of Corporation, 1975

Type of corporation	Rates not calculated	Personal Tax Rates <0	0-20	20-40	40-60	>60	Rates calculated
Public freq.	5	2	4	5	2	20	33
Col. %		15.38	50.00	50.00	25.00	32.26	
Row %		6.06	12.12	15.15	6.06	60.61	
Subsidiaries freq.	8	4	3	2	3	18	30
Col. %		30.77	37.50	20.00	37.50	29.03	
Row %		13.33	10.00	6.67	10.00	60.00	
Private freq.	11	2	0	2	1	5	10
Col. %		15.38	0.00	20.00	12.50	8.06	
Row %		20.00	0.00	20.00	10.00	50.00	
Interlisted freq.	5	5	1	1	2	19	28
Col. %		38.46	12.50	10.00	25.00	30.65	
Row %		17.86	3.57	3.57	7.14	67.86	
Total	29	13	8	10	8	62	101
Proportion		12.87	7.92	9.90	7.92	61.39	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	2.486	with 6 D.O.F.	PROB = 0.8701
Cramer's V	0.111		
Chi-square likelihood ratio	2.474	with 6 D.O.F.	PROB = 0.8714

Table A-7

Distribution of Individual Tax Rates
by Type of Corporation, 1976

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public							
freq.	6	4	5	2	7	14	32
Col. %		28.57	100.00	16.67	33.33	25.93	
Row %		12.50	15.63	6.25	21.88	43.75	
Subsidiaries							
freq.	4	1	0	6	7	20	34
Col. %		7.14	0.00	50.00	33.33	37.04	
Row %		2.94	0.00	17.65	20.59	58.82	
Private							
freq.	9	4	0	1	2	5	12
Col. %		28.57	0.00	8.33	9.52	9.26	
Row %		33.33	0.00	8.33	16.67	41.67	
Interlisted							
freq.	5	5	0	3	5	15	28
Col. %		35.71	0.00	25.00	23.81	27.78	
Row %		17.86	0.00	10.71	17.86	53.57	
Total	24	14	5	12	21	54	106
Proportion		13.21	4.72	11.32	19.81	50.94	

Test of effect of control type on inter-category rates distribution
(after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	9.534	with 6 D.O.F.	PROB = 0.1457
Cramer's V	0.212		
Chi-square likelihood ratio	11.235	with 6 D.O.F.	PROB = 0.0814

Table A-8

Distribution of Individual Tax Rates by Type of Corporation, 1977

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public							
freq.	4	1	1	3	16	13	34
Col. %		7.69	33.33	25.00	41.03	29.55	
Row %		2.94	2.94	8.82	47.06	38.24	
Subsidiaries							
freq.	3	7	1	2	14	11	35
Col. %		53.85	33.33	16.67	35.90	25.00	
Row %		20.00	2.86	5.71	40.00	31.43	
Private							
freq.	7	3	0	1	4	6	14
Col. %		23.08	0.00	8.33	10.26	13.64	
Row %		21.43	0.00	7.14	28.57	42.86	
Interlisted							
freq.	5	2	1	6	5	14	28
Col. %		15.38	33.33	50.00	12.82	31.82	
Row %		7.14	3.57	21.43	17.86	50.00	
Total	19	13	3	12	39	44	111
Proportion		11.71	2.70	10.81	35.14	39.64	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	6.919	with 6 D.O.F.	PROB = 0.3284
Cramer's V	0.177		
Chi-square likelihood ratio	7.098	with 6 D.O.F.	PROB = 0.3119

Table A-9

Distribution of Individual Tax Rates
by Type of Corporation, 1978

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public							
freq.	4	4	4	1	9	16	34
Col. %		33.33	66.67	14.29	30.00	32.00	
Row %		11.76	11.76	2.94	26.47	47.06	
Subsidiaries							
freq.	10	4	2	3	8	11	28
Col. %		33.33	33.33	42.86	26.67	22.00	
Row %		14.29	7.14	10.71	28.57	39.29	
Private							
freq.	6	0	0	0	4	11	15
Col. %		0.00	0.00	0.00	13.33	22.00	
Row %		0.00	0.00	0.00	26.67	73.33	
Interlisted							
freq.	5	4	0	3	9	12	28
Col. %		33.33	0.00	42.86	30.00	24.00	
Row %		14.29	0.00	10.71	32.14	42.86	
Total	25	12	6	7	30	50	105
Proportion		11.43	5.71	6.67	28.57	47.62	

Test of effect of control type on inter-category rates distribution
(after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	7.672	with 6 D.O.F.	PROB = 0.2631
Cramer's V	0.191		
Chi-square likelihood ratio	9.891	with 6 D.O.F.	PROB = 0.1293

Table A-10

Distribution of Individual Tax Rates by Type of Corporation, 1979

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public							
freq.	3	6	1	5	11	12	35
Col. %		37.50	100.00	29.41	35.48	23.08	
Row %		17.14	2.86	14.29	31.43	34.29	
Subsidiaries							
freq.	4	3	0	6	9	16	34
Col. %		18.75	0.00	35.29	29.03	30.77	
Row %		8.82	0.00	17.65	26.47	47.06	
Private							
freq.	2	5	0	1	3	10	19
Col. %		31.25	0.00	5.88	9.68	19.23	
Row %		26.32	0.00	5.26	15.79	52.63	
Interlisted							
freq.	4	2	0	5	8	14	29
Col. %		12.50	0.00	29.41	25.81	26.92	
Row %		6.90	0.00	17.24	27.59	48.28	
Total	13	16	1	17	31	52	117
Proportion		13.68	0.85	14.53	26.50	44.44	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	7.972	with 6 D.O.F.	PROB = 0.2436
Cramer's V	0.184		
Chi-square likelihood ratio	8.347	with 6 D.O.F.	PROB = 0.2138

Table A-11

Distribution of Individual Tax Rates by Type of Corporation, 1980

Type of corporation	Rates not calculated	Personal tax rates <0	0-20	20-40	40-60	>60	Rates calculated
Public							
freq.	5	4	1	1	12	15	33
Col. %		28.57	50.00	14.29	37.50	25.86	
Row %		12.12	3.03	3.03	36.36	45.45	
Subsidiaries							
freq.	5	5	0	3	9	16	33
Col. %		35.71	0.00	42.86	28.13	27.59	
Row %		15.15	0.00	9.09	27.27	48.48	
Private							
freq.	4	3	1	0	4	9	17
Col. %		21.43	50.00	0.00	12.50	15.52	
Row %		17.65	5.88	0.00	23.53	52.94	
Interlisted							
freq.	3	2	0	3	7	18	30
Col. %		14.29	0.00	42.86	21.88	31.03	
Row %		6.67	0.00	10.00	23.33	60.00	
Total	17	14	2	7	32	58	113
Proportion		12.39	1.77	6.19	28.32	51.33	

Test of effect of control type on inter-category rates distribution (after regrouping Categories 1 and 2, as well as 3 and 4)

Chi-square	3.878	with 6 D.O.F.	PROB = 0.6931
Cramer's V	0.131		
Chi-square likelihood ratio	4.045	with 6 D.O.F.	PROB = 0.6705

APPENDIX B

Distribution of Inferred Tax Rates in a Fixed Sample of Corporations, 1970-80

Year	Personal tax rate (per cent)						
	<0	0-25	25-50	50-75	75-100	>100	Total
1970	8	3	7	7	10	21	56
1971	9	5	10	9	5	18	56
1972	12	8	7	9	12	8	56
1973	7	4	12	11	5	17	56
1974	16	6	6	13	11	4	56
1975	11	8	14	9	7	7	56
1976	7	7	12	21	8	1	56
1977	7	8	17	21	3	0	56
1978	9	4	15	21	5	2	56
1979	11	7	12	17	5	4	56
1980	9	2	9	15	10	11	56

APPENDIX C

Table C-1

Analysis of Differences between Successive Distributions of Inferred Rates (Chi-Square, 5 Degrees of Freedom)

	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
1970	2.89	8.33	4.02	15.31*	11.48*	25.35*	34.17*	24.53*	18.56*	5.84
1971		7.48	0.69	13.34*	6.01	19.17*	23.27*	16.71*	10.79	5.53
1972			9.19	2.71	3.36	12.27*	21.15*	14.24*	7.23	5.74
1973				14.63*	6.32	16.84*	20.38*	13.88*	9.86	4.39
1974					6.11	8.71	17.43*	9.84	5.17	7.16
1975						9.35	13.02*	8.46	3.42	6.97
1976							3.75	2.17	3.39	11.62*
1977								3.76	6.02	19.71*
1978									2.18	9.88
1979										7.56
1980										

* Indicates a significant Chi-Square at the 5 per cent level.

Table C-2

Analysis of Differences between Successive Distributions of Adjusted Ratios (Chi-Square, 5 Degrees of Freedom)

	1971	1972	1973	1974	1975	1976	1977	1978	1979	1980
1970	3.15	4.93	8.44	3.19	4.93	5.65	6.76	5.46	5.28	4.99
1971		1.86	4.99	3.10	3.83	3.86	4.30	2.21	2.65	4.56
1972			4.48	3.45	5.60	8.49	10.14	4.10	5.25	3.44
1973				1.74	9.23	6.77	10.29	2.59	4.06	4.03
1974					7.43	5.97	8.67	3.00	2.20	1.55
1975						3.65	6.26	2.78	9.13	12.47*
1976							1.32	1.85	6.54	11.37*
1977								4.73	7.80	12.85*
1978									4.08	7.33
1979										3.13
1980										

* Indicates a significant Chi-Square at the 5 per cent level.

NOTES

1 See Miller and Scholes (1982) and Hess (1982) for an analysis of the major contributions in this area.

2 See Elton and Gruber (1970), Booth and Johnston (1984), Lakonishok and Vermaelen (1983), as well as Harris, Roenfeldt, and Cooley (1982), and Auerbach (1981).

3 For a more complete analysis of the Canada Income Tax Act, see Gagnon and Suret (1985).

4 See Gagnon and Suret (1985).

5 Originally formulated by Kalay (1982).

6 Relation (13) is obtained by incorporating into equation (1) the transaction costs arising from stock sales on the last cum-dividend day and the first ex-dividend day, and then rearranging the various terms. Thus, in this context, relation (1) becomes:

$$Pc - QPc - t_g[Pc - Po - Q(Pc + Po)]$$

$$= Px - QPx - t_g[Px - Po - Q(Px + Po)] + D\,(1 - t_d).$$

7 For a detailed description of the data, see Booth and Johnston (1984).

8 These data were obtained from the "Laval Tape" described by Morgan and Turgeon (1978).

9 The mathematical relationship between b_1 and t_{pf} when t_g is assumed greater than zero is derived from manipulations of equation (17). The result is:

$$t_{pf} = \frac{1 + b_1 + \alpha}{(1 + t_{pp})(b_1/2 + \gamma)} \quad \text{(if } L = 0.5\text{)}.$$

The actual figures involved appear in Table 5.

10 This function has the following characteristics:

$$\text{(i)} \qquad \lim_{b_1 \to -2\gamma^+} \left[\frac{1 + b_1 + \alpha}{(1 + t_{pp})(b_1/2 + \gamma)} \right] = -\infty$$

$$\text{(ii)} \quad \lim_{b_1 \to -2\gamma^-} \left[\frac{1 + b_1 + \alpha}{(1 + t_{pp})(b_1/2 + \gamma)} \right] = + \infty$$

$$\text{(iii)} \quad \lim_{b_1 \to +\infty} \left[\frac{1 + b_1 + \alpha}{(1 + t_{pp})(b_1/2 + \gamma)} \right] = \frac{2}{(1 + t_{pp})}$$

$$\text{(iv)} \quad \frac{dt_{pf}}{db_1} = \frac{2\gamma - 1 - \alpha}{2(1 + t_{pp})(b_1/2 + \gamma)^2} > 0, \ \forall b_1 \neq -2\gamma$$

11 The mathematical relationship between b_1 and t_{pf} when $t_g = 0$ is expressed as follows:

$$t_{pf} = \frac{1 + b_1 + \alpha}{\gamma (1 + t_{pp})}.$$

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